Glacier complementarity

How to apply equations for where equations apply

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Outline

1. the views of two precise glaciologists
2. complementarity for glaciers
3. complementarity from optimization
4. consequences for modelers (*and real scientists too*)

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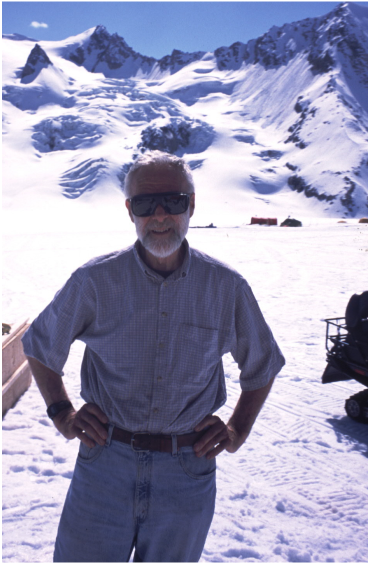
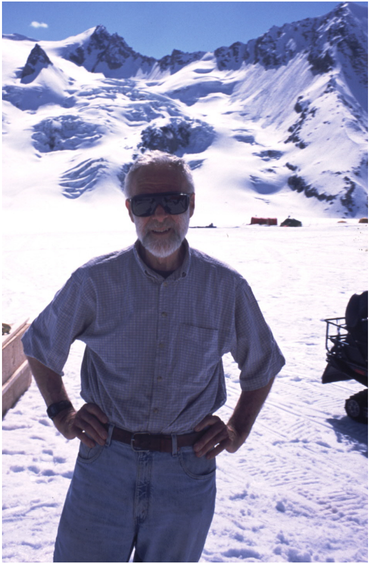


photo by Martin Truffer

- W is a glaciologist

W



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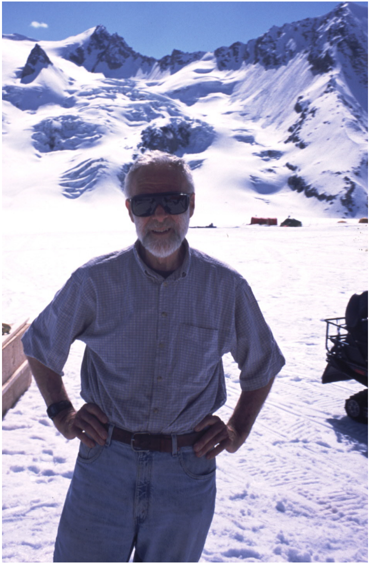

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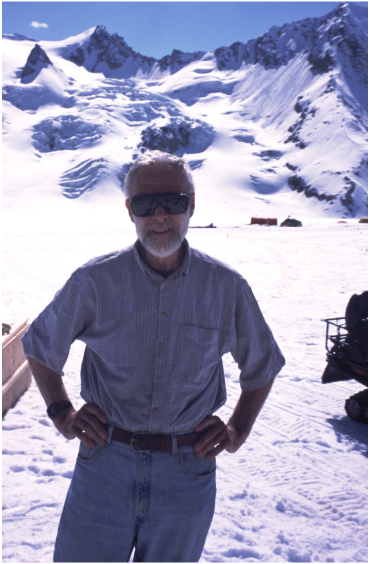

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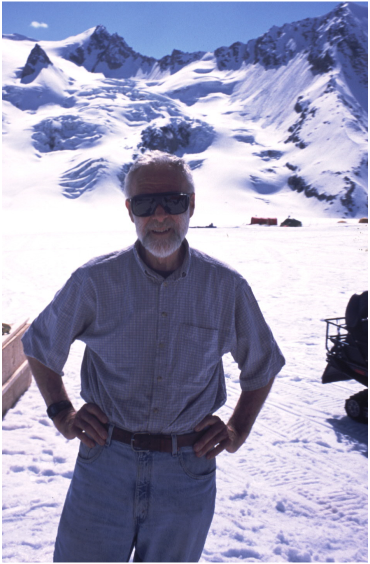

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$>$ the glacier thickness is positive

$$H > 0$$

$=$ the mass of ice is conserved

$$\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U} H) - a = 0$$

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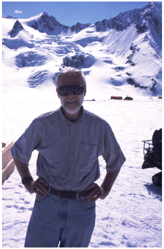
= the glacier thickness is zero

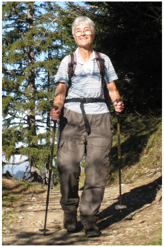
$$H = 0$$

> the annual surface mass balance is negative

$$-a > 0$$

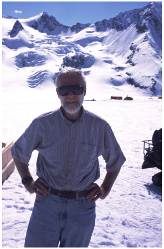
two views in different patches

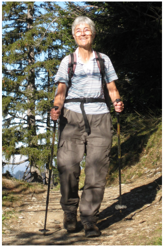




- *W says:* where I am
  the glacier thickness is positive
  $$H > 0$$
  and mass of ice is conserved
  $$\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U} H) - a = 0$$
- *A says:* where I am
  the glacier thickness is zero
  $$H = 0$$
  and the annual surface mass balance is negative
  $$-a > 0$$
- *a skeptic says:* so what? the world looks different in different places!

two views in different patches





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but first . . . define your terms

- $\mathbf{x} = (x, y)$ is map-plane location
- t is time

- $H(t, \mathbf{x})$ is glacier thickness
- $b(\mathbf{x})$ is bed elevation (not changing)
- $s(t, \mathbf{x})$ is glacier surface elevation
- $a(t, \mathbf{x})$ is annual surface (climatic) mass balance
  - a.k.a. the accumulation-ablation function
- $\mathbf{U} = \mathbf{U}(t, \mathbf{x})$ is vertically-averaged horizontal ice velocity
- $\mathbf{u} = \mathbf{u}(t, x, y, z)$ is ice velocity in 3D

the symbols are dumb, questions about them are not!

both views at once

- both W's view and A's view arise from one set of equations:

$$\begin{aligned} H &\geq 0 \\ \frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U}H) - a &\geq 0 \\ H\left(\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U}H) - a\right) &= 0 \end{aligned}$$

  - *consider*: W is standing on a glacier
  - *consider*: A is walking on a dirt trail

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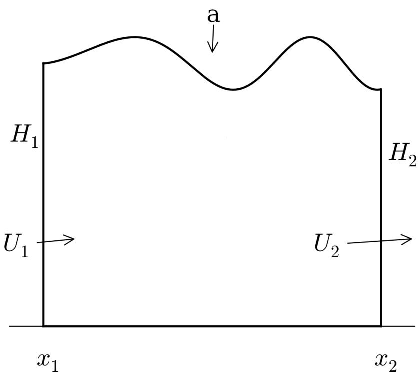
- "equations" will mean "set of equations and inequalities"
- the third equation is complementarity
- the whole thing is a (nonlinear) complementarity problem, an NCP

a reminder

- the main differential equation in this talk is the "mass conservation" or "continuity" equation

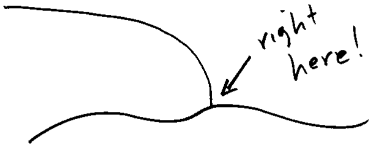
$$\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U} H) - a = 0$$



- the simple one horizontal dimension case is shown above
- flow into a segment of the glacier [$-\nabla \cdot (\mathbf{U} H)$], plus mass added at the top [a], determines rise or fall of the top: $\frac{\partial H}{\partial t} = -\nabla \cdot (\mathbf{U} H) + a$
- we may include basal motion and mass balance but this complicates the equation without changing my points

Outline

what is true at a glacier margin?



- one switches from W's view to A's view at a glacier margin
- both views are contained in the NCP:

$$H \geq 0$$

$$\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U}H) - a \geq 0$$

$$H\left(\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U}H) - a\right) = 0$$

- perhaps an extra equality holds at a margin:

$$H = 0 \qquad \textit{and} \qquad \frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{U}H) - a = 0$$

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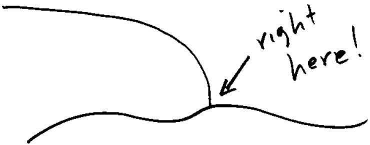

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useful to think about *open sets*

- "W is on a glacier" means a neighborhood of glacier is around W
- "A is on a trail" means a neighborhood of dirt is around A
- "neighborhood" $=$ open set in the map-plane

- *idea*: any strong-form differential equation, including an NCP using derivatives, only makes sense in open sets around locations

- but a glacier margin has no neighborhood of differentiability of H or $\mathbf{U}$,
- ...so the NCP equations do not hold *at* the margin

velocity or flux?

- . . . and in the NCP you might be worried about the zen question:
  what is the velocity $\mathbf{U}$ *of a glacier that isn't there?*

- let $\mathbf{q}$ be the map-plane *mass flux*; it *is* defined everywhere
- $\mathbf{q} = \mathbf{U}H$ *on the glacier*, but $H = 0$ and $\mathbf{q} = \mathbf{0}$ outside the glacier
  - $\mathbf{U} = \frac{\mathbf{q}}{H}$?
- mass conservation equation says $\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} = a$ *on the glacier*
- dynamics determines flow from geometry, so: $\mathbf{q} = \mathbf{q}(H)$
  - we can agree?: $\mathbf{q}(0) = \mathbf{0}$

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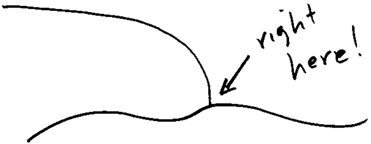

- both $H(t, \mathbf{x})$ and $\mathbf{q}(t, \mathbf{x})$ are continuous
  - caveat: . . . in any fluids view of glaciers where thickness is well-defined
  - violated by fracture and/or overhang
- so $H = 0$ *and* $\mathbf{q} = \mathbf{0}$ at a glacier margin
  - true whether the margin is advancing, stationary, or retreating
- the quantity $\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} - a$ actually jumps discontinuously
  - from zero on the glacier to substantially negative off the glacier

- *reminder*: NCP = nonlinear complementarity problem
- the glacier NCP applies *everywhere on Earth*:

$$
\begin{aligned}
H &\ge 0 \\
\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} - a &\ge 0 \\
H \left(\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} - a \right) &= 0
\end{aligned}
$$

  - at the South Pole 1000 years ago
  - 1000 years from now in the middle of Death Valley
  - outside my door right now

  - *except* right at glacier margins

applies everywhere, including where the glacier is

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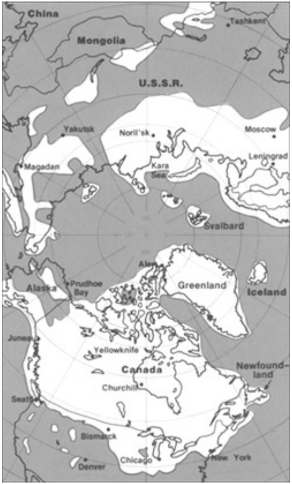
where be glaciers?

- "where is there a glacier?" is a first-class problem in glaciology
  - *example*: determine ice sheet extent in hypothesized previous climate
- we want our theory and models to apply everywhere, so that we may answer this first-class problem *within* a model
- an NCP is such a model:

$$\begin{aligned} H &\geq 0 \\ \frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} - a &\geq 0 \\ H\left(\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} - a\right) &= 0 \end{aligned}$$

  - it remains to compute $\mathbf{q}$ from geometry
  - . . . via conservation of momentum



J. Schlee
pubs.usgs.gov/gip/continents/

my main point

- if you say “my coupled climate-glacier model conserves mass” then think

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  everywhere, and not just

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  on the glacier

- note H, $\mathbf{q}$, a can be defined everywhere

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mass conservation or surface kinematical equation?

- *recall*: H is thickness, s surface elevation, and b bed elevation
  - $H = s - b$
  - $H_t = s_t$ because we have assumed $b_t = 0$
- the mass conservation equation and the surface kinematical equation (SKE) are equivalent if the ice is incompressible

$$\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} - a = 0 \qquad \iff \qquad \frac{\partial s}{\partial t} - \mathbf{u} \cdot \mathbf{n}_s - a = 0$$

  - where $\mathbf{n}_s = \left\langle -\frac{\partial s}{\partial x}, -\frac{\partial s}{\partial y}, 1 \right\rangle$ is normal to the surface

justification in the nonsliding and nonmelting base case, using Leibniz rule and incompressibility:

$$\begin{aligned}\nabla_{\mathbf{x}} \cdot \mathbf{q} &= \nabla_{\mathbf{x}} \cdot \left(\int_b^s \langle u, v \rangle \, dz \right) = \langle u, v \rangle \big|_s \cdot \nabla_{\mathbf{x}} s - \langle u, v \rangle \big|_b \cdot \nabla_{\mathbf{x}} b + \int_b^s \nabla_{\mathbf{x}} \cdot \langle u, v \rangle \, dz \\ &= \langle u, v \rangle \big|_s \cdot \nabla_{\mathbf{x}} s - \int_b^s w_z \, dz = \langle u, v \rangle \big|_s \cdot \nabla_{\mathbf{x}} s - w\big|_s = -\mathbf{u} \cdot \mathbf{n}_s\end{aligned}$$

- in sliding and/or melting base cases the **SKE remains the same** but the mass conservation equation is modified

NCP using the surface kinematical equation

- the SKE does not care about incompressibility or the basal motion
- restated NCP:

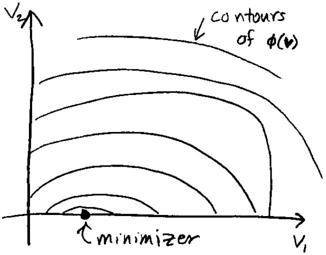
$$\begin{aligned} s - b &\geq 0 \\ \frac{\partial s}{\partial t} - \mathbf{u} \cdot \mathbf{n}_s - a &\geq 0 \\ (s - b)\left(\frac{\partial s}{\partial t} - \mathbf{u} \cdot \mathbf{n}_s - a\right) &= 0 \end{aligned}$$

- SKE form is a bit better and more general
- not really more fundamental
  - either NCP form assumes H and s are well-defined, thus no overhangs, and this *is* a fundamental restriction on glacier geometry

Outline

- I did not invent "complementarity"
- optimization problem:

$$\min_{\mathbf{v}\in\mathbb{R}^n} \phi(\mathbf{v}) \qquad \text{subject to} \qquad \mathbf{v} \ge 0$$



- Lagrange multipliers:

$$\Phi(\mathbf{v},\boldsymbol{\lambda}) = \phi(\mathbf{v}) - \boldsymbol{\lambda}\cdot\mathbf{v}$$

then

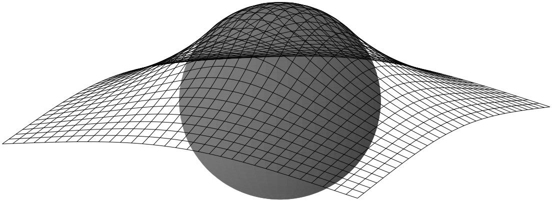
$$\mathbf{v} \ge 0, \qquad \boldsymbol{\lambda} \ge 0, \qquad \nabla\phi(\mathbf{v}) - \boldsymbol{\lambda} = 0, \qquad \mathbf{v}\,\boldsymbol{\lambda} = 0$$

  - these are KKT conditions
  - last condition "$\mathbf{v}\,\boldsymbol{\lambda} = 0$" is complementary slackness

- eliminate $\boldsymbol{\lambda}$ and define $\mathbf{F}(\mathbf{v}) = \nabla\phi(\mathbf{v})$ to state as NCP:

$$\begin{aligned} \mathbf{v} &\ge 0 \\ \mathbf{F}(\mathbf{v}) &\ge 0 \\ \mathbf{v}\,\mathbf{F}(\mathbf{v}) &= 0 \end{aligned}$$

example and analogy: obstacle problem



- the well-known *obstacle problem* is an ∞-dimensional NCP
  - just like the glacier problem
  - it is *also* constrained optimization
- membrane position $u = u(\mathbf{x})$ solves:

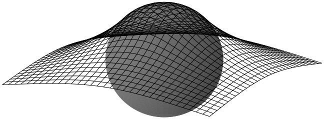
$$\min_v \phi(v) \qquad \text{subject to} \qquad v \ge \psi$$

where

$$\phi(v) = \int_\Omega \frac{1}{2}|\nabla v|^2 - f\,v\,d\mathbf{x}$$

  - in the figure, Ω is a square, ψ is the grey upper hemisphere, $f = 0$, and the solution u is shown as a mesh
  - u and v live in a space of functions on Ω
    - $v \in H^1_g(\Omega)$ where g gives boundary values

no one wants a variational inequality?



- equivalent obstacle problem formulations:

CO $\qquad u = \min_v \phi(v) \quad \text{s.t. } v \ge \psi$

VI $\qquad \int_\Omega \nabla u \cdot \nabla(v - u)\, d\mathbf{x} \ge \int_\Omega f(v-u)\, d\mathbf{x} \quad \text{for all } v \ge \psi$

NCP
$$\begin{aligned} u - \psi &\ge 0 \\ -\nabla^2 u - f &\ge 0 \\ (u-\psi)(-\nabla^2 u - f) &= 0 \end{aligned}$$

- VI = *variational inequality*
- I've concluded nobody really thinks VI style
- NCPs are easier to understand, both for scientists and mathematicians
- CO = *constrained optimization* is pretty intuitive, but ...

the glacier problem is not constrained optimization

- unfortunately, glaciers do not optimize any energy functional
  - no such energy has been offered
  - in known theories the implied symmetry is missing
- obstacle problem:

$$\text{CO} \leftrightarrow \text{VI} \leftrightarrow \text{NCP}$$

- glacier problem has no optimization form:

$$\text{VI} \leftrightarrow \text{NCP}$$

- NCP is a “strong form” (pointwise statements)
- CO and VI are “weak forms” (integrals, function spaces)
- I’m stuck thinking in, and explaining via, an NCP

how about all the other glacier equations?

- momentum/energy conservation equations only apply within the glacier
- for this talk their “purpose” is to provide velocity in the NCP:

$$\text{(geometry, boundary stress, thermal state)} \implies \mathbf{U}, \mathbf{q}, \mathbf{u}$$

- i.e.

$$\begin{aligned} s - b &\geq 0 \\ \frac{\partial s}{\partial t} - \mathbf{u}_{\text{from solving the mass/momentum/energy coupled problem}} \cdot \mathbf{n}_s - a &\geq 0 \\ (s - b)\left(\frac{\partial s}{\partial t} - \mathbf{u}_{\text{from solving the mass/momentum/energy coupled problem}} \cdot \mathbf{n}_s - a\right) &= 0 \end{aligned}$$

- my current project: solve NCP with isothermal Stokes dynamics in Firedrake/PETSc

steady state? implicit time steps?

- it is easy to state NCPs for these situations
- if a glacier is in steady state with a steady climate a then

$$\begin{aligned} s - b &\ge 0 \\ -\mathbf{u}\cdot\mathbf{n}_s - a &\ge 0 \\ (s-b)\,(-\mathbf{u}\cdot\mathbf{n}_s - a) &= 0 \end{aligned}$$

- if we want to solve for s after a backward Euler step of Δt then

$$\begin{aligned} s - b &\ge 0 \\ s - s_{\text{old}} - \Delta t\,(\mathbf{u}\cdot\mathbf{n}_s + a) &\ge 0 \\ (s-b)\,(s - s_{\text{old}} - \Delta t\,(\mathbf{u}\cdot\mathbf{n}_s + a)) &= 0 \end{aligned}$$

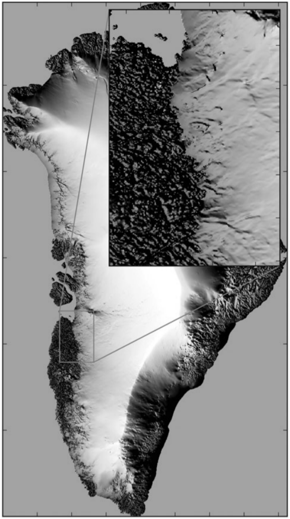
- well-posedness proofs in some particular cases
  - (treating NCP and VI forms as equivalent)
  - `arxiv.org/abs/2007.05625`
  - explicit steps have regularity issues (separate discussion)

Outline

reasons a numerical modeler should care

in your evolving glacier model:

1. the NCP *is* the source of margin shape
   - this is true in the continuum model, regardless of momentum balance
   - no need to impose a shape to the margin
2. the NCP is a sanity check
   - check each part of NCP once converged
   - *my experience*: it is worth measuring and/or fixing the violations which arise within one cell of the margin
3. you can run an NCP solver on a computer
   - software already exists
     - ▷ SNESVI and TAO in PETSc/TAO
       `www.mcs.anl.gov/petsc`
     - ▷ dune-solvers in DUNE
       `www.dune-project.org`
   - Bueler (2016) is a PETSc example
     - ▷ steady state SIA case (GIS at right)

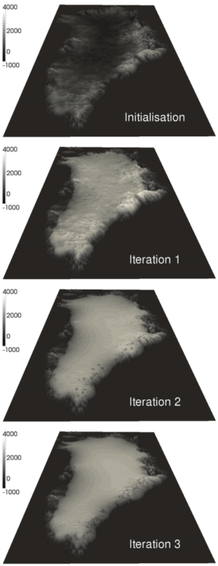


inside an NCP solver ("active set" method)

- for an implicit time-step:

$$\begin{aligned} s - b &\geq 0 \\ s - s_{\text{old}} - \Delta t \left(\mathbf{u} \cdot \mathbf{n}_s + a \right) &\geq 0 \\ (s - b) \left(s - s_{\text{old}} - \Delta t \left(\mathbf{u} \cdot \mathbf{n}_s + a \right) \right) &= 0 \\ [\text{momentum balance}] &= 0 \end{aligned}$$

- sketch of Newton iteration for one time-step:
  - a and b given as data on $\Omega \times [t, t + \Delta t]$
  - set initial iterate $s^{(0)} = s_{\text{old}}, \mathbf{u}^{(0)} = \mathbf{u}_{\text{old}}$
  - for $k = 1, 2, 3, \dots$
    - compute all residuals in NCP *and* momentum
    - for all $\mathbf{u}^{(k)}$ variables, and for $s^{(k)} > b$ or $s^{(k)} - s_{\text{old}} - \Delta t \left(\mathbf{u}^{(k-1)} \cdot \mathbf{n}_s^{(k-1)} + a \right) \leq 0$ variables, solve Newton step linear equations for search direction
    - get $s^{(k)}, \mathbf{u}^{(k)}$ by line search
    - repeat until tolerance
  - $s = s^{(k)}$ is updated surface elevation at $t + \Delta t$
- steady state is $\Delta t \to \infty$ extreme case
  - Jouvet & Bueler (2012) at right: DUNE, $s^{(0)} = 0$



discrete-time mass conservation fails

- for fixed-boundary glacier problems one has mass conservation exactly:

$$M_n = M_{n-1} + C_n$$

  - $M_n =$ (total ice mass at time t_n)
  - $C_n =$ (total SMB, i.e. climate input, applied to ice during $[t_{n-1}, t_n]$)
  - $t_n = t_{n-1} + \Delta t$
- in solving the implicit time-step NCP, if any margin *retreats* then such mass conservation *will fail*
- thus accounting for mass errors is needed
  - $M_n = M_{n-1} + C_n - R_n$ where R_n is retreat mass
  - area of retreat is unboundable in theory, but $R_n \to 0$ as $\Delta t \to 0$

- `arxiv.org/abs/2007.05625` de-mystifies this and the next slide

discrete-time mass conservation fails

- for fixed-boundary glacier problems one has mass conservation exactly:

$$M_n = M_{n-1} + C_n$$

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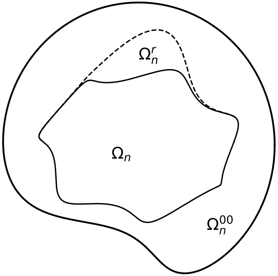
why does discrete-time mass conservation fail?

- *claim:* when solving the NCP, if any margin *retreats* then mass conservation *will fail*
- even if we solve the continuous-space NCP exactly
- why?
  - suppose ice extents Ω_{n-1} (old) and Ω_n (new)
  - consider the retreat area:

$$\begin{aligned}\Omega_n^r &= \{\mathbf{x} \,:\, s_{n-1}(\mathbf{x}) > b(\mathbf{x}) \;\&\; s_n(\mathbf{x}) = b(\mathbf{x})\} \\ &= \Omega_{n-1} \setminus \Omega_n\end{aligned}$$

  - question: *for* $\mathbf{x} \in \Omega_n^r$*, when did the ice thickness go to zero and how much surface mass balance, versus flow into* Ω_n^r*, was needed to do it?*
    - ▷ exact discrete mass conservation requires you know the answer
    - ▷ information to answer this question requires cutting up the time step



A's view as logic

- even on a dirt trail, precise glaciology is possible:

  If the glacier thickness here is zero then the annual surface mass balance cannot be positive.

- we already knew that? I hope so
- remember the glossary? (IACS 2011. *Glossary of glacier mass balance and related terms*)
- proposal?: define an *admissible annual (climatic) surface mass balance* (AACSMB) as any a, defined everywhere, which satisfies NCP

$$s - b \geq 0$$

$$\frac{\partial s}{\partial t} - \mathbf{u} \cdot \mathbf{n}_s - a \geq 0$$

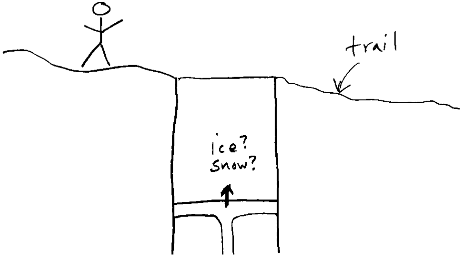
$$(s - b)\left(\frac{\partial s}{\partial t} - \mathbf{u} \cdot \mathbf{n}_s - a\right) = 0$$

  - identifies SMB a as term in conservation equation (✓) *and* requires compatibility for modeled SMB values *outside* the current glacier
  - your predictive ice dynamics model wants AACSMBs from climate models

surface mass balance should be modeled (almost) everywhere

- the above proposal is a joke, but . . .
- numerically-computed stable time steps for evolving glaciers require modeled surface mass balance *everywhere*, not just *on* the glacier
  - or at least "nearby" . . . otherwise GIGO
- this is practical
- for example, consider an energy balance model for bare ice or snow SMB
- requires a "thought experiment" in a dirt area:

  how fast would a piston of ice or snow need to rise in order to stay at trail level in this hypothesized climate?



are there big ideas in glaciology?

- I am not sure most glaciologists have an opinion on this
- I disagree
- some possible "big ideas":
  - volume-area scaling
  - hysteresis from elevation-dependent mass balance
  - the tidewater glacier cycle
  - why glaciers surge
- none of these are obvious even to smart non-glacier scientists
- add "complementarity gives glacier extent" to list?
  - viewpoint incipient in Bodvardsson 1955?
  - viewpoint fully present Calvo et al 2002

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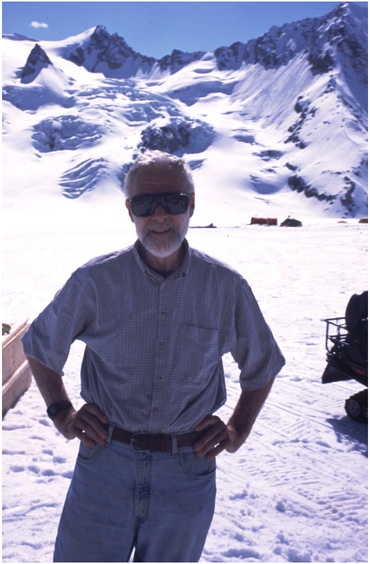
Almut Iken (1933–2018)



Will Harrison (1936–2020)